Closing Thu: Taylor Notes 4, 5
Final is Saturday, March 12
5:00-7:50pm, KANE 130
Eight pages covers everything

TN 5: Using Taylor Series

Idea: Manipulate the 6 series we now know to get other series. Within the interval of convergence, we can

1. Substitute (replace x by something else)

2. Integrate; note that
$$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C$$

3. Differentiate; note that
$$\frac{d}{dx}(x^n) = nx^{n-1} + C$$

4. Combine; note that
$$\sum_{k=0}^{\infty} kx^k - 3\sum_{k=0}^{\infty}\frac{1}{k!}x^k = \sum_{k=0}^{\infty}\left(k - \frac{3}{k!}\right)x^k$$

Here are the 6 series we know and can quote:
Recall: For the following we have the
open interval of convergence: -∞ < x < ∞

$$e^x = \sum_{k=0}^{\infty}\frac{1}{k!}x^k$$

$$\sin(x) = \sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)!}x^{2k+1}$$

$$\cos(x) = \sum_{k=0}^{\infty}\frac{(-1)^k}{(2k)!}x^{2k}$$

And for these we have the
open interval of convergence: **-1 < x < 1**

$$\frac{1}{1-x} = \sum_{k=0}^{\infty}x^k$$

$$-\ln(1-x) = \sum_{k=0}^{\infty}\frac{1}{k+1}x^{k+1},$$

$$\arctan(x) = \sum_{k=0}^{\infty}\frac{(-1)^k}{2k+1}x^{2k+1}$$

Substitution Questions

Find the Taylor series based at b = 0 (and give the interval of convergence for each) of

(a) $f(x) = 3e^{2x}$

(b) $g(x) = \frac{5}{1-4x}$

(c) $h(x) = \frac{3}{2x+1}$

Combining and Working with sums

Find the Taylor series based at b=0 (and give the interval of convergence for each) of

(a) $y = 7 + 3x^5 e^{2x}$

(b) $y = \frac{5}{1-4x} - \frac{3}{2x+1}$

(c) $y = \cos^2(x)$ (Big hint: Half-angle)

Integrating

(a) Give the first three nonzero terms of the Taylor Series for

$$\int_0^x 7 + 3t^5 e^{2t}\, dt$$

(b) Find a Taylor series for (this is from HW):

$$A(x) = \int_0^x \frac{\sin(t)}{t}\, dt$$

1. What is A'(x)?
2. Give the Taylor series for sin(t) based at 0.
3. What can you say about sin(t)/t?
4. Integate.